

Significantly super-Chandrasekhar limiting mass white dwarfs and their consequences

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Abstract.

Since 2012, we have initiated a new idea showing that the mass of highly magnetized or modified Einstein's gravity induced white dwarfs could be significantly super-Chandrasekhar with a different mass-limit. This discovery has several important consequences, including explanation of peculiar, over-luminous type Ia supernovae, soft gamma-ray repeaters and anomalous X-ray pulsars without invoking extraordinarily strong, yet unobserved, magnetic fields. It further argues for a possible second standard candle. Based on simpler calculations, these white dwarfs are also shown to be much less luminous than their standard counter-parts (of low magnetic fields). This discovery altogether initiates a new field of research.

1. Introduction

Since 2012, we have initiated exploring highly magnetized super-Chandrasekhar white dwarfs (B-WDs). The primary aim behind this was explaining peculiar, over-luminous type Ia supernovae (SNeIa). However, subsequently they were found to be useful to explain other data, e.g. soft gamma-ray repeaters (SGRs) and anomalous X-ray pulsars (AXPs), white dwarf (WD) pulsar(s), etc. This immediately brings the topic super-Chandrasekhar WDs in lime-light, with so many groups' coming forward to work in this new field, who need not be focusing on magnetic effects only (just to mention a very few out of the bulk, Liu et al. 2014; Franzon & Schramm 2015; Belyaev et al. 2015). In order to establish this field, our approach has been, so far, the following.

First, we have considered most simplistic, spherically symmetric, highly magnetized B-WDs in the Newtonian framework. This brings quantum mechanical effects in the equation of state (EoS), in the presence of high amplitude of field (Das & Mukhopadhyay 2012, 2013). In the same model, we have also shown that B-WDs altogether have a new mass-limit, 80% larger than the Chandrasekhar-limit, in the same spirit as the Chandrasekhar-limit was obtained (Chandrasekhar 1935). Afterwards, we removed the assumptions of Newtonian description and spherical symmetry (e.g. Das

& Mukhopadhyay 2014a, 2015a). Based on a full scale general relativistic magnetohydrodynamic (GRMHD) description (Das & Mukhopadhyay 2015a; Subramanian & Mukhopadhyay 2015), we explored more self-consistent B-WDs that are ellipsoids and have revealed similar masses as obtained in the simpler framework.

In a different avenue, we also explored modified Einstein's gravity (Starobinsky model) induced model (Das & Mukhopadhyay 2015b) to unify under- and over-luminous SNeIa. We would not touch this work in the limited scope of this proceedings.

To follow the motivation of this work and systematic progress of the topic in detail, the readers are advised to see Mukhopadhyay 2015, and the references therein (also see, Ostriker & Hartwick 1968). Here we touch upon the basic results.

2. Effect of magnetic field via equation of state

We assume the magnetic field to be fluctuating in such a way that effective field brings negligible effect over matter pressure. Hence, they do not contribute to MHD. However, the fluctuating length scale is larger than the Compton wavelength of electrons (λ_e) so that Landau quantization affects the degenerate electron gas EoS, given by

$$P = \frac{2B_D}{(2\pi)^2 \lambda_e^3} m_e c^2 \sum_{\nu=0}^{\nu_m} g_\nu (1 + 2\nu B_D) \eta \left(\frac{x_F(\nu)}{(1 + 2\nu B_D)^{1/2}} \right), \quad (1)$$

$$\eta(y) = \frac{1}{2} y \sqrt{1 + y^2} - \frac{1}{2} \ln(y + \sqrt{1 + y^2}),$$

where $B_D = B/4.414 \times 10^{13} \text{ G}$. Other variables have their usual meanings, see Mukhopadhyay 2015, for details. At high density (e.g. around the center of star), EoS approximately reduces to $P = K_m(B) \rho^2$.

The underlying magnetized, spherical B-WD obeys the magnetostatic equilibrium condition, along with estimate of mass, as

$$\frac{1}{\rho + \rho_B} \frac{d}{dr} \left(P + \frac{B^2}{8\pi} \right) = F_g + \frac{\vec{B} \cdot \nabla \vec{B}}{4\pi(\rho + \rho_B)} \Big|_r, \quad \frac{dM}{dr} = 4\pi r^2 (\rho + \rho_B). \quad (2)$$

For the present purpose, magnetic terms could be neglected in above equations and following Lane-Emden formalism (Das & Mukhopadhyay 2013), the scalings of mass and radius with central density ρ_c are obtained as

$$M \propto K_m^{3/2} \rho_c^{(3-n)/2n}, \quad R \propto K_m^{1/2} \rho_c^{(1-n)/2n}, \quad K_m = K \rho_c^{-2/3}. \quad (3)$$

Clearly $n = 1$ ($\Gamma = 2$) corresponds to ρ_c -independent M (unlike the Chandrasekhar's case when K_m is independent of B and limiting mass corresponds to $n = 3$).

Substituting proportionality constants appropriately, we obtain the limiting mass

$$M_l = \left(\frac{hc}{2G} \right)^{3/2} \frac{1}{(\mu_e m_H)^2} \approx \frac{10.312}{\mu_e^2} M_\odot, \quad (4)$$

when the limiting radius $R_l \rightarrow 0$. For $\mu_e = 2$, $M_l = 2.58 M_\odot$. Importantly, for finite but high density and magnetic field, e.g. $\rho_c = 2 \times 10^{10} \text{ gm/cc}$ and $B = 8.8 \times 10^{15} \text{ G}$ when $E_{Fmax} = 20 m_e c^2$, $M = 2.44 M_\odot$ and R is about 650 km. Note that these ρ_c and B are below their respective upper limits set by the instabilities of pycnonuclear fusion, inverse- β decay and general relativistic effects (Das & Mukhopadhyay 2014b).

3. Effect of magnetic field via MHD

When magnetic field is not fluctuating, magnetic pressure and density cannot be neglected in the stellar structure equations. We model such B-WDs using the publicly available *XNS* code (Pili et al. 2014). In Fig. 1 we present some representative results, reported in detail earlier (Das & Mukhopadhyay 2015a; Subramanian & Mukhopadhyay 2015; Mukhopadhyay 2015), showing again that B-WDs could be significantly super-Chandrasekhar. Note however that we restrict the central field in such a way that the ratio of magnetic to gravitational energies is significantly below unity. This restriction furthermore hinders EoS to be modified by Landau quantization.

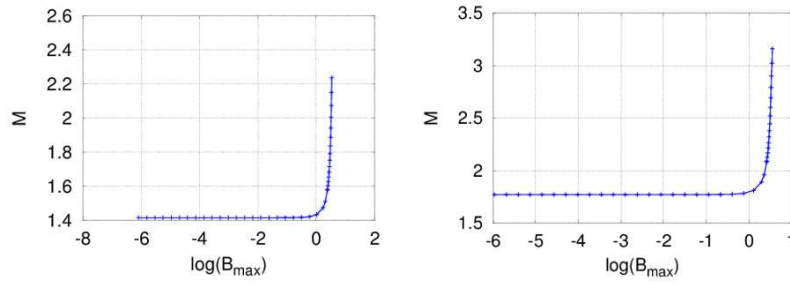


Figure 1. Non-rotating (left) and rotating (right) sequences of mass in M_\odot with changing maximum field in G, for toroidal magnetic fields. Chosen central angular velocity is 30.42 rad/sec. See Subramanian & Mukhopadhyay 2015, for details.

4. Luminosity of B-WDs

Now following an established technique (Shapiro & Teukolsky 1983), we divide B-WDs into inner core having degenerate EoS (for the present purpose non-relativistic) and outer envelope having ideal gas EoS. We then solve the magnetostatic equilibrium and photon diffusion equations in the presence of a magnetic field given by

$$\frac{d}{dr}(P + P_B) = -\frac{GM}{r^2}(\rho + \rho_B) \quad \text{and} \quad \frac{dT}{dr} = -\frac{3}{4ac} \frac{\kappa_0(\rho + \rho_B)^2}{T^{6.5}} \frac{L}{4\pi r^2}, \quad (5)$$

with $B\left(\frac{\rho}{\rho_0}\right) = B_s + B_0 \left[1 - \exp\left(-\eta\left(\frac{\rho}{\rho_0}\right)^\gamma\right)\right]$, $\rho_* \approx (2.4 \times 10^{-8} \text{ g cm}^{-3}) \mu_e T_*^{3/2}$,

and investigate the temperature profile in the envelope. Here B_s is the surface magnetic field and B_0 is similar to the central field B_c , the value of ρ_0 is chosen to be 10% of ρ_c , also ρ_* and T_* are the interface density and temperature respectively. We also fix surface density $\rho_s = 10^{-9} \text{ gm/cc}$ and radius $R = 5000 \text{ km}$ and solve above equations simultaneously, for non-magnetic WDs and B-WDs separately. Interestingly, for fixed interface radius and/or temperature between magnetic and non-magnetic cases, B-WDs turn out to be much less luminous, as seen in Table 1. This is roughly understood, in this simplistic model, from the magnetostatic balance condition for similar mass and radius between two cases. Generally for B under consideration, $\rho_B \ll \rho$ but $P \sim \rho_B$. Hence,

for a similar gravitational field, B-WDs have to have a smaller thermal energy and hence luminosity (L). As $L < 10^{-5}L_{\odot}$ cannot be detected yet, B-WDs with $L \lesssim 10^{-6}L_{\odot}$, as given in Table 1, appear to be invisible.

Table 1. Variation of luminosity with magnetic field for a fixed interface radius. For other details, see Bhattacharya et al. 2015.

$B = (B_s, B_c)$ (in G)	L (in L_{\odot})	T_* (in K)	ρ_* (in g cm^{-3})	T_s (in K)
$B = (0, 0)$	10^{-5}	2.332×10^6	170.722	3850
$B = (10^9, 10^{13})$	5.17×10^{-6}	1.94346×10^6	129.886	3260
$B = (10^9, 5 \times 10^{13})$	2.87×10^{-8}	495107	16.7012	890
$B = (10^{10}, 5 \times 10^{12})$	1.35×10^{-6}	1.37451×10^6	77.2534	2330
$B = (10^{11}, 10^{13})$	2.33×10^{-13}	70750.2	0.902173	50

5. B-WDs as SGRs/AXPs

SGRs/AXPs are popularly explained by magnetar model (Duncan & Thompson 1992). However, there are many shortcomings in the magnetar model, see Mereghetti 2012, for details. Weakly magnetized WD based model is challenged by observed short spin periods and low UV-luminosities (L_{UV}). We explore the possibility to explain the high energy phenomena in AXPs/SGRs by rotationally powered magnetic energy (\dot{E}_{dot}) of B-WDs – there is no need to invoke extraordinary, yet observationally unconfirmed, sources of energy. This is possible because B-WDs have larger moment of inertia than neutron stars, which is however small enough to produce L_{UV} .

Figure 2 shows that \dot{E}_{rot} computed based on B-WD model, with a fixed inclination angle between rotation and magnetic axes $\alpha = 15^\circ$, is several orders of magnitude larger than observed X-ray luminosity L_x for nine sources.

6. Are GCRT J1745-3009 and AR Sco B-WDs?

The transient radio source GCRT J1745-3009 was argued earlier to be a WD pulsar (Zhang & Gil 2005), but this idea was ruled out by color-magnitude analysis (Kaplan et al. 2008). Now in the framework of a very slowly rotating B-WD with $B_s \sim 3.3 \times 10^{11} - 2 \times 10^{12}$ G, corresponding $R \sim 1580 - 500$ km and $\rho_c \sim 10^{10}$ gm/cc, we revisit all the calculations, e.g. radius of polar cap and unipolar potential drop therein, etc., done by Zhang & Gil (2005), and find them to be consistent with WD pulsar idea (see Mukhopadhyay & Rao 2016, for details). The maximum gamma-ray/X-ray flux appears to be only a factor of 4 larger than that obtained earlier (Zhang & Gil 2005) for the same parameters for very highly magnetized B-WDs. The condition of radio luminosity not exceeding the spin-down luminosity reveals the distance of the source to be $\lesssim 8.5$ kpc, which is in accordance with the lower limit predicted previously (Kaplan et al. 2008). As shown in §4, such a B-WD is significantly cooler, hence its optical flux will be dimmer to evade detection, strongly supporting it to be a WD pulsar.

Recently, AR Sco has been found to be a WD pulsar with spin period 1.95 min (Marsh et al. 2016). While a WD following Chandrasekhar's mass-radius relation

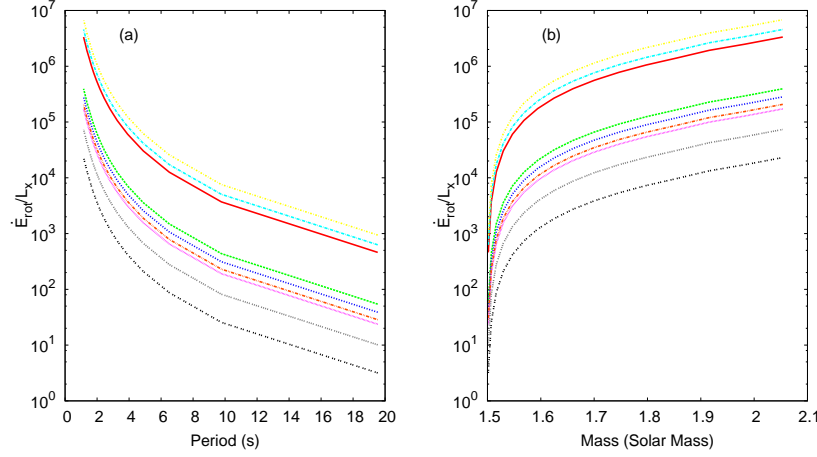


Figure 2. The ratio of rate of rotational energy release to X-ray luminosity as a function of (a) spin period, and (b) mass, for B-WDs when from the top to bottom various curves correspond to 1E 1547-54, 1E 1048-59, SGR 1806-20, SGR 1900+14, SGR 0526-66, SGR 1822-1606, 1E 1841-045, SGR 0418+5729 and 1E 2259+586. For other details, see Mukhopadhyay & Rao 2016.

with mass $0.8 - 1.29M_{\odot}$ could explain the source, in order to explain its emission as spin-down power, its B_s could be $\sim 10^9$ G and hence B_c could be 10^{12} G. However, if eventually accretion process starts in AR Sco (e.g. by WD's coming closer to the companion by the emission of gravitational radiation), based on our past result (Das et al. 2013), initial smaller (B_s, B_c) may enhance via flux-freezing and WD may deviate from Chandrasekhar's mass-radius path leading to a B-WD. Hence, AR Sco is plausibly a seed of B-WD.

7. Critique of B-WDs

Since the birth of this field, while several groups have been supporting it with follow-up work (e.g. Liu et al. 2014; Franzon & Schramm 2015; Belyaev et al. 2015, to mention a very few), there are some critics as well. While some criticism (Nityananda & Konar 2014) was found to be based on erroneous calculations and was dismissed immediately (Das & Mukhopadhyay 2015c), some others (Chamel et al. 2013; Maleiro et al. 2012) were shown to be misleading (Das & Mukhopadhyay 2014b; Mukhopadhyay 2015). Recently, a new issue has been brought up (Bera & Bhattacharya 2016), arguing that such magnetized WDs are unstable. However, this argument is based on purely poloidal (or highly poloidally dominated) and purely toroidal field configurations, which have long been proposed to be unstable (Tayler 1973, also see the comments in Das & Mukhopadhyay 2015a). They are not expected to be naturally occurring configurations. On a technical note, some of the non-linear perturbation analysis carried out by Bera & Bhattacharya (2016) seems to show the non-perturbed equilibrium state itself to be evolving with time, which also raises a question regarding the robustness of the method

employed. Moreover, B-WDs could have fluctuating fields and, hence, overall low average fields, as discussed in section 2, which do not encounter such a problem, if any. We believe that one should look for plausible mixed field configurations (Ciolfi & Rezzolla 2013) with self-consistent inclusion of rotation, to perform such a stability analysis, before drawing any bold conclusions.

8. Conclusion

We have initiated a new field establishing the possible formation of highly super-Chandrasekhar magnetized WDs, which will also have a new mass-limit. Such WDs have several important implications: formation of peculiar overluminous SNeIa, SGRs/AXPs, WD pulsars, to name a few. Our future aim is to systematically unfold all the issues related to such WDs, including self-consistent stability analysis based on realistic magnetic field geometries, exploring their actual connection to over-luminous SNeIa etc. Hence, we welcome the community to join us to work in this fascinating new field.

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